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Two-loop renormalization group analysis of hadronic decays of a charged Higgs boson

Andrzej Czarnecki[1]

Department of Physics, University of Alberta,
Edmonton, Canada T6G 2J1

and

Andrei I. Davydychev[2]

Department of Physics, University of Bergen,
Allégaten 55, N-5007 Bergen, Norway

Abstract

We calculate next-to-leading QCD corrections to the decay $H^+ \to u\bar{d}$ for generic up and down quarks in the final state. A recently developed algorithm for evaluation of massive two-loop Feynman diagrams is employed to calculate renormalization constants of the charged Higgs boson. The origin and summation of large logarithmic corrections to the decay rate of the top quark into a lighter charged Higgs boson is also explained.

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[1]Address after September 30th, 1993: Institut für Physik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany

[2]On leave from Institute for Nuclear Physics, Moscow State University, 119899, Moscow, Russia. E-mail address: davyd@vsfys1.fi.uib.no

1 Introduction

The accelerators planned to be built in the near future will provide an insight into physics at TeV energy scale and thus probe a region especially interesting from the point of view of electroweak interactions. Therefore we observe recently an increased interest in various aspects of phenomenology of the electroweak symmetry breaking, in the framework of the Standard Model and its extensions, which predict one or more doublets of Higgs bosons.

As far as the experimental detection of the Higgs particles is concerned, detailed knowledge of their decay properties is of special interest. In the present paper we concentrate on hadronic decays of charged Higgs bosons, predicted e.g. by the Minimal Supersymmetric Model (see ref. [1] for a review and further references). In the case of the Standard Model Higgs boson hadronic decays have been analyzed in a number of publications. QCD and QED corrections were first calculated in [2], where it was noted that as the ratio of mass of the decaying scalar particle to that of fermions in the final state increases, the one-loop corrections diverge logarithmically. This problem was solved by renormalizing mass of quarks at the energy scale equal to the mass of the decaying particle and thus absorbing the large correction into the tree level decay rate expressed in terms of the running mass. This approach was subsequently extended by means of the renormalization group technique in ref. [3, 4] (see also ref. [1] for a clear summary), where next-to-leading QCD corrections were summed up in the case of large mass of the Higgs boson. The effect of three-loop QCD corrections was first calculated in ref. [5] and further analyzed in ref. [6]. Leading logarithmic approximation and various ways to parametrize the next-to-leading corrections to the Standard Model Higgs boson are subject of several recent studies [7, 8, 9].

In the minimal extension of the Standard Model suggested by supersymmetry there are two Higgs boson doublets, and one of the charged modes becomes physical. It is therefore of great importance to look at the phenomenology of such charged scalar particles, since their discovery would give information about the theory underlying the Standard Model. Various radiative corrections to hadronic decays of charged scalar particles have been calculated and published recently. One-loop QCD corrections and the sum of leading logarithms can be found in [10, 11] and the leading electroweak effects in the limit of large mass of the top quark in [12, 13] (see also [14]).

The purpose of the present paper is to apply the renormalization group technique to calculate next-to-leading QCD corrections to the process $H^+ \to u\bar{d}$, where u and d represent generic up and down type quarks respectively. In section 2 we describe the theoretical framework of this calculation, and in 3 we present the calculation of the two-loop mass and wave function renormalization constants of a charged Higgs boson. The results for the corrected rate of the Higgs decay are presented in section 4. In section 5 we digress to discuss an application of the summing of leading logarithms to the closely related process $t \to H^+ b$. Our conclusions are given in section 6.

2 Decay rate and operator product expansion

As has been demonstrated in refs. [3, 4], the hadronic decay rate of a Higgs boson can be represented by [3]

$$\Gamma\left(H^+ \to u\bar{d}\right) \equiv \Gamma_H = \frac{g^2}{m_H}\,\mathrm{Im}\,C_0(q^2), \tag{1}$$

where C_0 is the coefficient of the unit operator in the operator product expansion of the correlator function of the scalar currents [4],

$$\mathrm{i}\int \mathrm{d}^D x\, e^{\mathrm{i}\,(qx)}\,\mathrm{T}\left[J_H(x)J_H(0)\right] = \sum_{d,l} C_d^l(q^2)\mathcal{O}_d^l, \tag{2}$$

and the scalar current in the present case is defined by

$$J_H = Z_1'\,\frac{1}{2m_W}\,\bar{u}\,(aR + bL)\,d. \tag{3}$$

In this formula Z_1' denotes the renormalization constant of the charged Higgs–fermion vertex, R and L are the chiral projection operators, $R = (1+\gamma_5)/2$ and $L = (1-\gamma_5)/2$, and the coefficients a and b depend on the specific model. We will consider two models characterized by the absence of flavour changing neutral currents, described in detail in [1], where references to original papers can also be found. In model I we have

$$a = \sqrt{2}\,m_u \cot\beta, \qquad b = -\sqrt{2}\,m_d \cot\beta, \tag{4}$$

whereas in model II, which corresponds to the Higgs sector of the Minimal Supersymmetric Standard Model:

$$a = \sqrt{2}\,m_u \cot\beta, \qquad b = \sqrt{2}\,m_d \tan\beta, \tag{5}$$

and $\tan\beta$ is the ratio of vacuum expectation values of the two Higgs doublets.

We now describe the procedure of calculating next-to-leading QCD contributions to the coefficient function $C_0(q^2)$. Following ref. [3] and using methods described in [16] one derives the renormalization group equation for the space-like values of the argument of C_0 ($q^2 < 0$). The solution to this equation is found in form of an expansion in the QCD coupling constant g_s and in the ratios of masses $m_{u,d}/m_H$ (below we shall also use a common notation for the quark masses, $m_q \equiv m_{u,d}$). Keeping only first two terms in the mass expansion is justified in the region far above the threshold of the $u\bar{d}$ production; near the threshold not only this expansion is insufficient, but also the perturbative treatment of the vacuum polarization diagrams is questionable.

While the general method follows closely ref. [3] and needs no further discussion here, we want to concentrate on the novel feature of the present calculation, namely on the computation of the renormalization constants of the charged Higgs boson.

[3]We shall omit "plus" in notations related to H^+; for example, $m_H \equiv m_{H^+}$, $\Gamma_H \equiv \Gamma_{H^+}$, etc.

[4]The sum on the r.h.s. of (2) goes over the indices d and l, where d denotes the canonical dimension of a local operator $\mathcal{O}_d^l$ while l labels independent operators with the same d; there is only one (unit) operator with $d = 0$: $\mathcal{O}_0 = 1$.

3 Derivation of renormalization constants

We work in $D = 4 - \omega$ dimensional space, considering γ_5 to be anticommuting with other γ-matrices, $\gamma_5^2 = 1$. We may use such a scheme because there is no anomaly problem in the case considered (see, e.g., ref. [15]). Solution of the renormalization group equation requires knowledge of $1/\omega$ poles of the quantities

$$S = g^{-2}(1 - Z_3') \qquad \text{and} \qquad T = g^{-2}(1 - Z_{m_H} Z_3'), \tag{6}$$

with Z_3' and Z_{m_H} denoting wave function and mass renormalization constants of the charged Higgs field. Hence we have to calculate divergent parts of the self energy diagrams as depicted in figure 1. It has to be noted that in the present case of two different masses of quarks in the loop, diagrams corresponding to figs. 1(b) and 1(d) should be considered together with their counterparts with corrections on the other quark line. Their sum is of course symmetric under $m_u \leftrightarrow m_d$.

A modification of the method developed in ref. [17] enabled us to obtain exact expressions for divergent parts of all diagrams presented in figure 1, that are valid for any values of the external momentum k, m_u and m_d. Since in the present paper we are mainly interested in the expansion in m_q^2/m_H^2, we expand the relevant integrals in m_q^2/k^2, keeping the k^2 and the $k^2(m_q^2/k^2) = m_q^2$ terms only. In the one-loop order (see figure 1(a)) we get

$$-\frac{\mathrm{i}g^2 N_C}{(4\pi)^2 2m_W^2 \omega}\left\{4abm_u m_d + (a^2+b^2)\left(2(m_u^2+m_d^2) - k^2\right)\right\}, \tag{7}$$

while the sum of all two-loop-order contributions (see figures 1(b,c,d,e) together with counterparts) yields

$$\frac{\mathrm{i}g^2 g_s^2 N_C C_F}{(4\pi)^4 4m_W^2 \omega}\left\{(a^2+b^2)\left[\frac{12}{\omega}\left(4(m_u^2+m_d^2) - k^2\right) + \left(5k^2 - 8(m_u^2+m_d^2)\right)\right]\right.$$
$$\left. +16abm_u m_d\left(\frac{6}{\omega} - 1\right)\right\}. \tag{8}$$

In the above formulae the colour factors are $N_C = 3$ and $C_F = 4/3$. It is remarkable that the terms containing $\ln(m_q^2)$ and $\ln(-k^2)$ (occurring in the expressions for separate diagrams) disappear in the whole sum (8). In particular, this fact enables us to consider analytic continuation to time-like values of the momentum without difficulty. It should be also noted that the factors of $\pi^{-\omega/2}\Gamma(1+\frac{1}{2}\omega)$ are included into the definition of coupling constants g^2 and g_s^2, as it is usually done in the framework of the $\overline{MS}$ scheme (this is also equivalent to a re-definition of $1/\omega$ poles).

We can now calculate coefficients S_n and T_n of $1/\omega^n$ poles in S and T including terms of the order of g_s^2:

$$S_1 = \frac{N_C}{(4\pi)^2}\,\frac{a^2+b^2}{2m_W^2}\left(1 + \frac{g_s^2}{(4\pi)^2}\,s^{(1)}\right), \tag{9}$$

$$T_1 = \frac{N_C}{(4\pi)^2}\,\frac{2abm_u m_d + (a^2+b^2)(m_u^2+m_d^2)}{m_W^2 m_H^2}\left(1 + \frac{g_s^2}{(4\pi)^2}\,t^{(1)}\right), \tag{10}$$

$$S_2 = -\frac{N_C}{(4\pi)^2}\,\frac{a^2+b^2}{m_W^2}\,\frac{4g_s^2}{(4\pi)^2}, \tag{11}$$

$$T_2 = -\frac{N_C}{(4\pi)^2}\,\frac{2abm_u m_d + (a^2+b^2)(m_u^2+m_d^2)}{m_W^2 m_H^2}\,\frac{16g_s^2}{(4\pi)^2}, \tag{12}$$

with $s^{(1)} = \frac{10}{3}$ and $t^{(1)} = \frac{8}{3}$. We have displayed S_2 and T_2 because they illustrate nice agreement of our calculation with equations following from the renormalization group analysis [3, 16]. Namely, both these quantities can be found in the lowest relevant order of perturbation theory from [5]

$$S_2 = \frac{g_s^2}{(4\pi)^2}\,\gamma_m^{(0)}\,S_1 + O(g_s^4), \qquad T_2 = 2\frac{g_s^2}{(4\pi)^2}\,\gamma_m^{(0)}\,T_1 + O(g_s^4), \tag{13}$$

and we reproduce our expressions by putting $\gamma_m^{(0)} = -8$.

4 Corrected decay width

Here we shall present the formula for the decay rate of the charged Higgs boson including next-to-leading corrections. We define $\mathcal{L} = \ln\left(m_H^2/\Lambda_{\rm QCD}^2\right)$ and obtain:

$$\Gamma_H = \frac{N_C g^2 m_H}{32\pi m_W^2}\,\mathcal{L}^{\gamma_m^{(0)}/\beta_0} \times \left\{\frac{\hat{a}^2+\hat{b}^2}{2}\left[1 + \frac{\beta_1\gamma_m^{(0)}}{\beta_0^3}\,\frac{\ln\mathcal{L}}{\mathcal{L}} + \frac{1}{\beta_0\mathcal{L}}\left(2s^{(1)} - 2\gamma_m^{(0)} + \frac{1}{\beta_0}\left(\frac{\beta_1}{\beta_0}\gamma_m^{(0)} - \gamma_m^{(1)}\right)\right)\right] + \delta\right\}, \tag{14}$$

where the mass correction δ is

$$\delta = \frac{\mathcal{L}^{\gamma_m^{(0)}/\beta_0}}{m_H^2}\left\{\frac{2\gamma_m^{(0)}}{\beta_0\mathcal{L}}\left(4\hat{a}\hat{b}\widehat{m}_u\widehat{m}_d + (\hat{a}^2+\hat{b}^2)(\widehat{m}_u^2+\widehat{m}_d^2)\right) - \left(2\hat{a}\hat{b}\widehat{m}_u\widehat{m}_d + (\hat{a}^2+\hat{b}^2)(\widehat{m}_u^2+\widehat{m}_d^2)\right) \times \left[1 + \frac{2\beta_1\gamma_m^{(0)}}{\beta_0^3}\,\frac{\ln\mathcal{L}}{\mathcal{L}} + \frac{2}{\beta_0\mathcal{L}}\left(t^{(1)} + \frac{1}{\beta_0}\left(\frac{\beta_1}{\beta_0}\gamma_m^{(0)} - \gamma_m^{(1)}\right)\right)\right]\right\}. \tag{15}$$

We have expressed the decay width of the charged Higgs boson in terms of the renormalization group invariant masses of the quarks $\widehat{m}_q$ ($q = u, d$), and the coefficients $\hat{a}$ and $\hat{b}$ defined by equations (4) and (5) with m_q replaced by $\widehat{m}_q$.[6] Although eqs. (14)

[5] Here and below, $\gamma_m^{(n)}$ and β_n correspond to the coefficients of expansion (in g_s) of the anomalous dimension of mass, $\gamma_m(g_s)$, and the beta function, $\beta(g_s)$. In the normalization used (see, e.g., ref. [3] and references therein) we have: $\beta_0 = 11 - \frac{2}{3}N_F$, $\beta_1 = 102 - \frac{38}{3}N_F$, $\gamma_m^{(0)} = -8$, $\gamma_m^{(1)} = -\frac{404}{3} + \frac{40}{9}N_F$, where N_F is the number of quark flavours.

[6] If we formally put $\widehat{m}_u = \widehat{m}_d = \hat{a} = \hat{b} \equiv \widehat{m}_q$ in eqs. (14) and (15), we get the correct result for partial decay width of a neutral Higgs into $q\bar{q}$ pair (see eqs. (3.21)–(3.22) in [3]).

and (15) correspond to partial decay width $H^+ \to u\bar{d}$, it is clear that the main contribution to the sum over generations will be given by the term with maximal quark masses allowed.

Following the method of ref. [3, 18] we use the threshold condition stating that the running mass of the quark at the energy scale of production of a pair quark-antiquark is equal to half this energy; from this condition we obtain:

$$\widehat{m}_q^{-2} = m_q^{-2} \left(\ln \frac{4m_q^2}{\Lambda_{\rm QCD}^2} \right)^{\gamma_m^{(0)}/\beta_0} \times \left\{ 1 + \frac{\beta_1 \gamma_m^{(0)}}{\beta_0^3} \frac{\ln\ln(4m_q^2/\Lambda_{\rm QCD}^2)}{\ln(4m_q^2/\Lambda_{\rm QCD}^2)} + \frac{1}{\beta_0^2} \left(\frac{\beta_1}{\beta_0} \gamma_m^{(0)} - \gamma_m^{(1)} \right) \frac{1}{\ln(4m_q^2/\Lambda_{\rm QCD}^2)} \right\}. \quad (16)$$

We now want to visualize the magnitude of the leading and next-to-leading order corrections. The leading order correction can be obtained from equations (14) and (15) by dropping all the terms divided by $\mathcal{L} = \ln\left(m_H^2/\Lambda_{\rm QCD}^2\right)$. In the case of model II, our formulae in the leading logarithmic approximation reproduce the results of ref. [10], for both reactions $H^+ \to c\bar{s}$ and $H^+ \to t\bar{b}$. To our knowledge, the corrections in model I have not been analyzed so far even in the leading order. We present both the leading and the next-to-leading logarithmic corrections to the rate of the decay $H^+ \to t\bar{b}$ in fig. 2(a-c) for the value of $\tan\beta = 2$, for which this decay has similar rate in both models (we take $m_t = 150$ GeV and $m_b = 4.5$ GeV). It turns out that although the leading corrections decrease the expected width of the charged Higgs boson, the next-to-leading terms can increase it by a large factor, especially for the light mass of the Higgs. On the other hand, this effect may be an artifact of the choice of the threshold condition. The sensitivity of the result to this choice is shown in fig. 2, where we present the results using two different conditions: (a,b) $\overline{m}(4m^2) = m$ and (c) $\overline{m}(m^2) = m$ (where $\overline{m}$ denotes the running mass of the quark). The dependence on the initial condition has also been discussed in much detail in ref. [25].

Size of corrections also strongly depends on the value of $\tan\beta$, as can be seen in fig. 3. For this plot we have used the usual condition $\overline{m}(4m^2) = m$, and we see that for large $\tan\beta$, where the decay is dominated by the coupling proportional to the bottom quark mass, next-to-leading corrections slightly decrease the effect of the leading ones. Therefore, the effect of increasing the width mentioned above is due to the corrections to the mass of the top quark; this may be a signal of insufficiency of the expansion in m_q/m_H for lighter Higgs bosons. We are going to address this issue in future (see also ref. [19]).

5 On the decay $t \to H^+ b$

The same term in the Lagrangian which is responsible for the decay of the charged Higgs boson into quarks will also enable a sufficiently heavy top quark to decay into a bottom quark and an H^+. Both electroweak [20, 21] and QCD [22, 23, 24] corrections to this decay have been studied at the one-loop level. It has been found [24] that in the two Higgs doublet model predicted by supersymmetry the relative size of QCD corrections becomes large for growing values of $\tan\beta$. We would like to discuss this effect here in order to demonstrate that this large correction can be absorbed in the Born rate if one uses the running mass of the b quark[7], just like in the case of the Standard Model Higgs boson [2, 25].

For this purpose we compute the tree level decay rate in the limit of very large $\tan\beta$ and mass of the top quark:

$$\Gamma^{(0)}(t \to H^+ b) = \frac{G_F m_t^3}{8\sqrt{2}\pi}|V_{tb}|^2(1-\chi^2)\left[4+(1-\chi^2)\tan\beta\right]\epsilon^2, \tag{17}$$

where we have introduced the following notation for the ratios of relevant masses:

$$\epsilon = \frac{m_b}{m_t}, \qquad \chi = \frac{m_H}{m_t} \qquad (m_H \equiv m_{H^+}). \tag{18}$$

The first order QCD corrections, calculated in ref. [24], can be expressed in the limit of large $\tan\beta$ by:

$$\Gamma^{(1)}(t \to H^+ b) = \frac{\alpha_s}{6\pi}\frac{G_F m_t^3\,|V_{tb}|^2}{\sqrt{2}\pi}\left[(2G_+ - G_0)\tan^2\beta + 4G_-\right]\epsilon^2, \tag{19}$$

and the explicit formulae for the coefficient functions G_i can be found in ref. [24]. Here we only need terms of the order of $\ln\epsilon$:

$$\begin{aligned} G_+ &\to \frac{3}{4}(1-\chi^2)^2\ln\epsilon, \\ G_- &\to 3(1-\chi^2)\ln\epsilon, \\ G_0 &\to -\frac{3}{2}(1-\chi^2)^2\ln\epsilon. \end{aligned} \tag{20}$$

Using these expressions we can calculate the asymptotic value of first order corrections for large values of $\tan\beta$ and m_t:

$$\Gamma^{(1)} = \frac{2\alpha_s}{\pi}\ln\left(\frac{m_b^2}{m_t^2}\right)\Gamma^{(0)}. \tag{21}$$

We see that for $\alpha_s \approx 0.1$, $m_b = 4.5$ GeV and $m_t = 100$ GeV this correction is of the order of -40%, in agreement with diagrams presented in [24]. The size of corrections becomes even larger as the mass of the top quark increases, and eventually the one-loop corrected rate of decay becomes negative; such large corrections are a sign of a breakdown of the perturbation theory. However, it is possible to avoid the large corrections by renormalizing the mass of the b quark not on the mass-shell but at the energy scale characteristic to the process, which is the mass of the top quark. The running mass of the bottom quark at this energy is:

$$\overline{m}_b(m_t^2) = m_b\left(\frac{\ln\left(4m_b^2/\Lambda^2_{\rm QCD}\right)}{\ln\left(m_t^2/\Lambda^2_{\rm QCD}\right)}\right)^{12/(33-2N_F)}, \tag{22}$$

[7]One of the authors (A.C.) is grateful to the referee of Physical Review D for suggesting this and to Sacha Davidson for helpful discussions on this topic.

where $N_F = 6$ is the number of quark flavours, and we take $\Lambda_{\rm QCD} = 150$ MeV (in the $\overline{\rm MS}$ scheme). We can expand the above expression in a series in the coupling constant α_s, and we find that:

$$\overline{m}_b(m_t^2) \approx m_b \left(1 + \frac{\alpha_s}{\pi} \ln\left(\frac{m_b^2}{m_t^2}\right)\right). \tag{23}$$

It can now be seen from the formula (21) that for large $\tan\beta$ and m_t the one-loop corrected decay rate approaches the Born rate expressed in terms of the running b quark mass. In figure 4 we show the dependence of decay rate of the top quark on $\tan\beta$. We note that for large values of $\tan\beta$ the QCD corrected rate is not much different from the Born rate expressed in terms of the running b quark mass (22), and that we no longer face the problem of unreasonably large corrections.

On the other hand, for the small values of $\tan\beta$ the Born rate remains approximately unchanged when expressed in terms of the running b mass. The reason for this is that the dominant coupling of the quarks to the charged Higgs is $m_t \cot\beta$ in this region and mass of the b quark does not play any important role. The same can be said about the analysis of the top quark decay in the non-supersymmetric two Higgs doublet model, and it explains why no large logarithmic corrections were found there for any values of $\tan\beta$ [24].

6 Summary

We have found leading and next-to-leading order corrections to the decay width of the charged Higgs boson in the framework of two models. In the case of the leading corrections in the model motivated by supersymmetry we confirmed the previously published formulae [10]; the remaining results are new. For a heavy Higgs boson or for large values of $\tan\beta$ we found that the next-to-leading order corrections sizably decrease the effect of the leading order corrections, and increase the final result for the decay rate. We have also examined the process $t \to H^+ b$, explaining the origin of large logarithmic corrections found in a previous paper [24].

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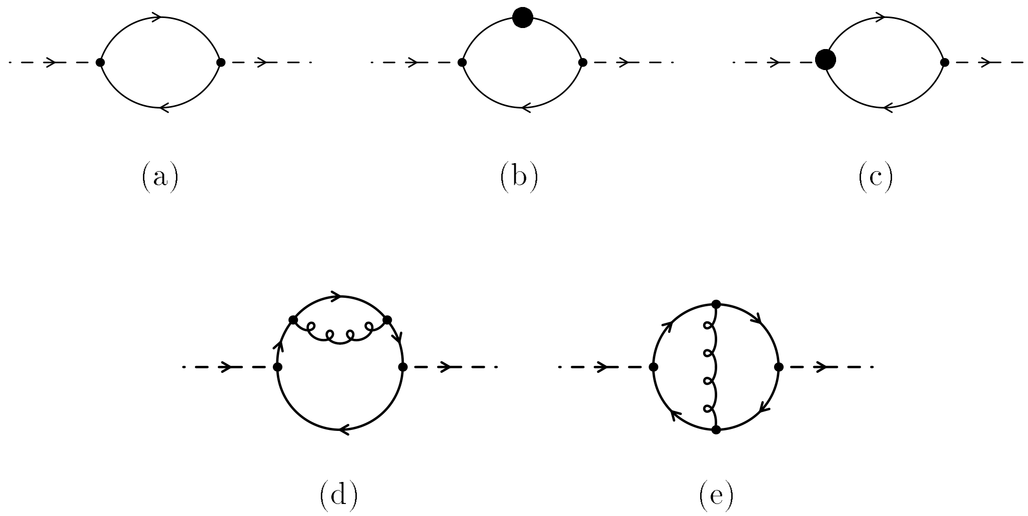
Figure captions

Figure 1: Types of diagrams contributing to the wave function and mass renormalization of the charged Higgs boson: one-loop diagram (a), quark propagator (b) and vertex (c) counterterms, and reducible (d) and irreducible (e) two-loop diagrams.

Figure 2: Rate of the charged Higgs decay (a) in Model II and (b,c) in Model I for two different threshold conditions (see text). Dashed: Born rate, dotted: leading, and solid: next-to-leading corrections.

Figure 3: Rate of the charged Higgs decay as a function of $\tan\beta$. Dashed: Born rate, dotted: leading, and solid: next-to-leading corrections.

Figure 4: Rate of the decay $t \to H^+ b$ for $m_t = 150$ GeV, $m_H = 80$ GeV, $m_b = 4.5$ GeV and $\alpha_s = 0.1$: Born rate (long dash), rate including first order QCD corrections (short dash), and the improved Born rate (solid line).



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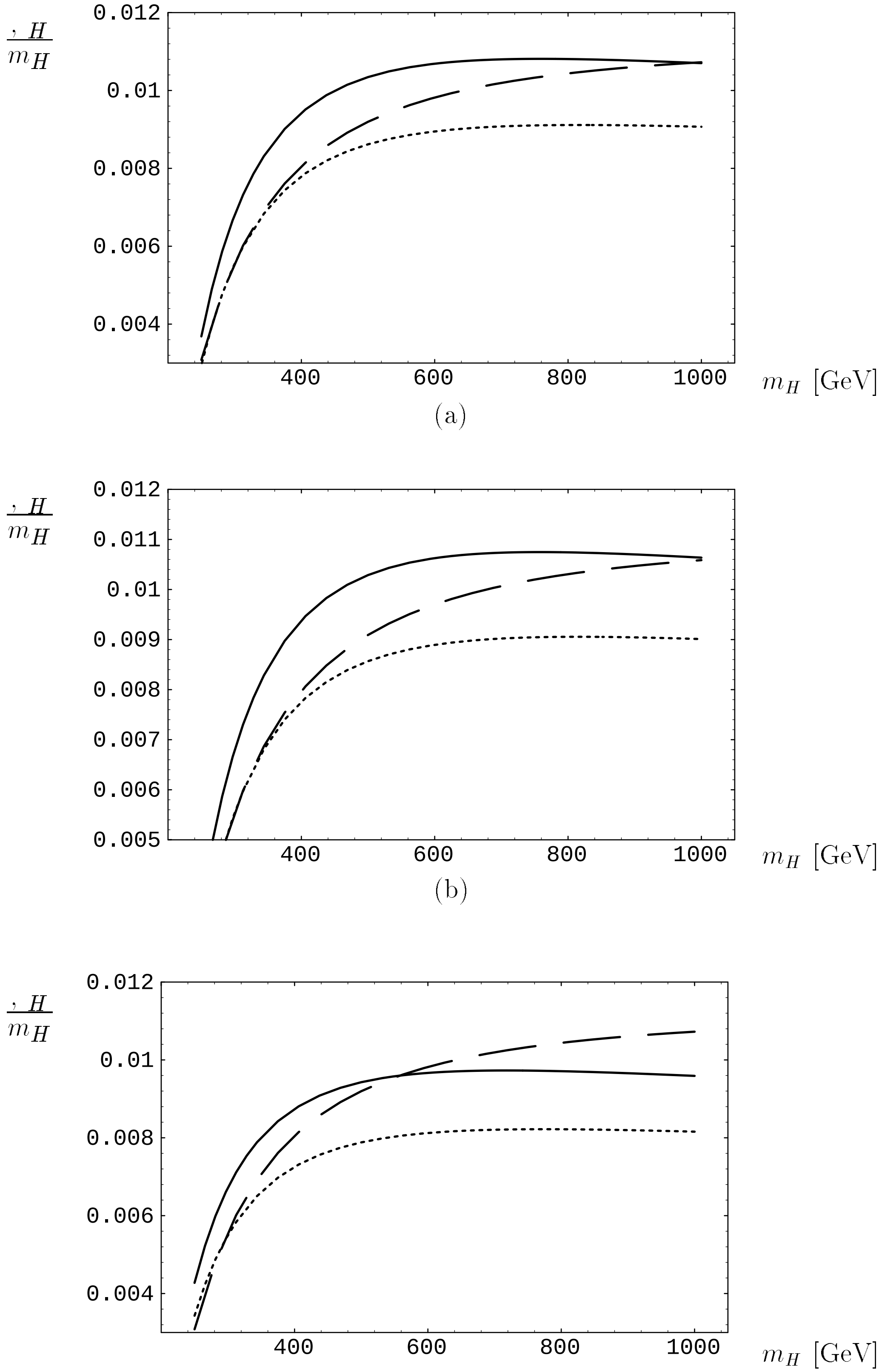

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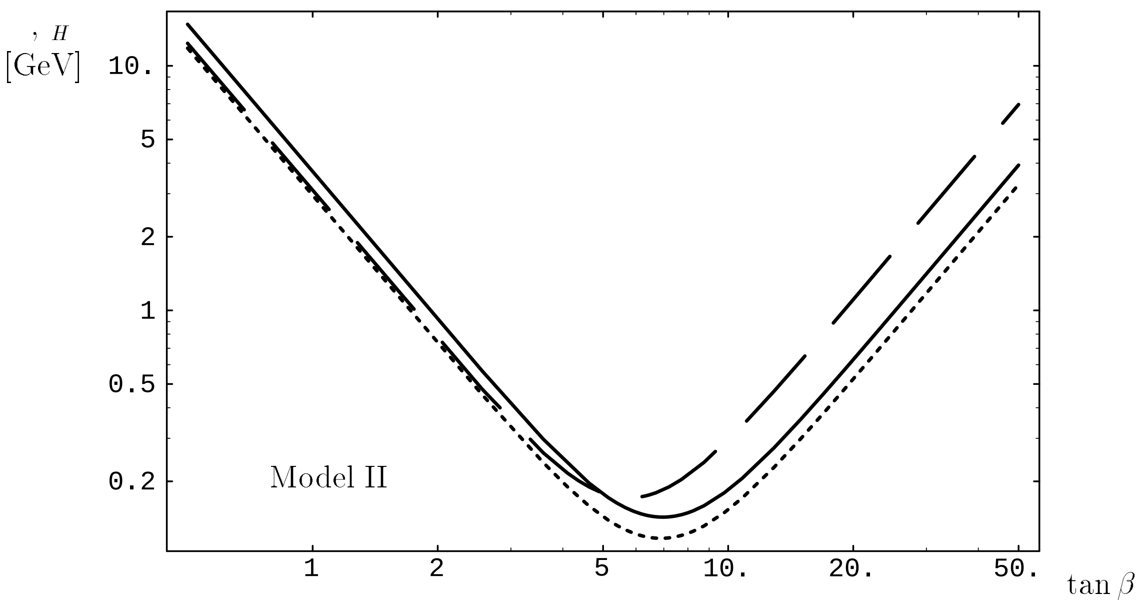

Figure 3: Rate of the charged Higgs decay as a function of $\tan\beta$. Dashed: Born rate, dotted: leading, and solid: next-to-leading corrections.

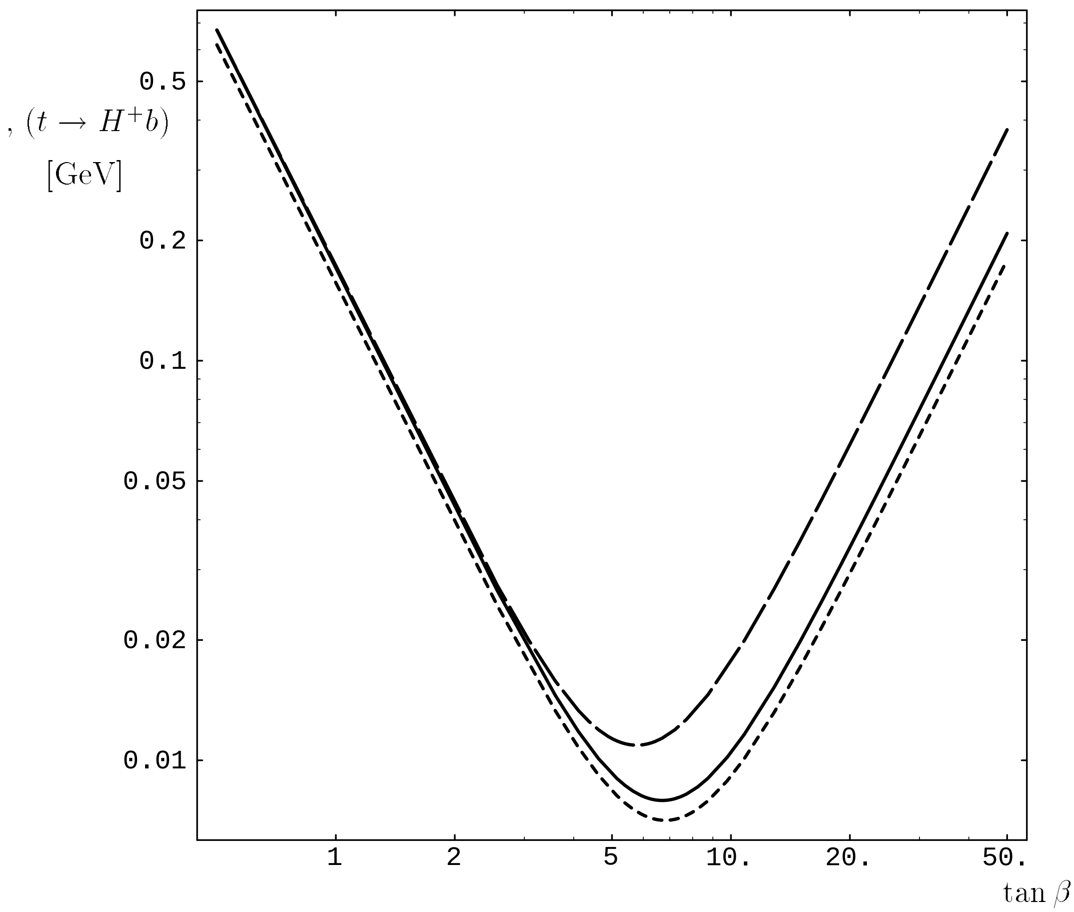

Figure 4: Rate of the decay $t \to H^+b$ for $m_t = 150$ GeV, $m_H = 80$ GeV, $m_b = 4.5$ GeV and $\alpha_s = 0.1$: Born rate (long dash), rate including first order QCD corrections (short dash), and the improved Born rate (solid line).